The Behaviour of Modern Flexible Framed Structures Undergoing Differential Settlement

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Abstract

Modern office buildings are often open plan buildings with a frame consisting of flat reinforced concrete (RC) slabs, RC columns and non-load bearing internal and external partitions and facades. These structures are more flexible than older conventional buildings with load bearing walls, and the use of conventional guidelines for differential settlement may therefore be over-conservative. Conventional guidelines focusing on 2D structures also lack provision for the 3D deformation of a structure. A numerical experiment was performed which consisted of the design of a 3D, 5-bay by 5-bay, six storey flat slab RC frame with pad foundations on clay. The behaviour of the designed structure undergoing differential settlement was predicted by means of linear-elastic finite element analysis. The results show it is possible to normalise the structural behaviour using a soil-structure stiffness ratio, and to demonstrate the importance of 3D deformation, and of linear vs. non-linear soil modelling.

Keywords: Soil-structure Interaction, Numerical Modelling,

1 Introduction

Differential settlement in structures is important because differential settlement of foundations often leads to damage within the structure (Burland et al., 2001a). The impact of differential settlement on a structure has been widely investigated since the late 1940’s (Meyerhof, 1947; Skempton and Macdonald, 1956; Polshin and Tokar, 1957; Grant et al., 1974; Burland and Wroth, 1975; Burland et al., 1977; Jardine et al., 1986; Potts and Addenbrooke, 1997; Potts et al., 1998, Burland et al., 2001a). This previous research provides valuable insight into the behaviour of structures undergoing differential settlement. However it focuses on older conventional type structures and is based on simplified 2D approaches.

The design of modern office buildings is often significantly different from the design of older buildings, resulting in a more flexible structure. Case studies for the Jubilee Line Extension in London (Burland et al., 2001b) show that load bearing brickwork was used in earlier buildings (1750 to 1950) and in lower (5-storeys or less) buildings of 1960 to 1990. Steel framed buildings mostly dated from 1900 to 1930, whilst reinforced concrete frame buildings date from 1915 to 1990. Even though the survey was limited, it shows a move away from load bearing brick infill (especially in higher buildings) towards the use of reinforced concrete
frames. Modern open plan offices with flat concrete slabs, concrete columns and glass facades are significantly more flexible than conventional load bearing brickwork structures and guidelines for conventional structures may therefore be overly conservative.

Burland and Wroth (1975) and Burland et al. (1977) developed a simplified method to predict damage due to differential settlement in buildings. The method was subsequently refined and is described in detail in Burland et al. (2001a). It was used successfully on the Jubilee Line Extension project. However it is important to realise the simplifications and limitations of the method, which assumes that the deformation of the structure is predominantly two dimensional and that it behaves like a beam. It was thought that these assumptions might no longer be true for modern flexible framed structures.

The advance in modern computers and modern finite element software packages allows for the analysis of 3D models with increased complexity. Simplified 2D models may provide valuable insight and use fewer resources; however full 3D models can include more detail and show any shortcomings of simplified 2D models. The research presented in this paper makes use of the capability to analyse a 3D structure to investigate the behaviour of modern flexible framed structures undergoing differential settlement.

2 Research Methodology

Finite element analysis was used to investigate the behaviour of modern flexible framed structures undergoing differential settlement. Modern office buildings vary in design and to minimise the numerical modelling a “typical” modern structure was investigated. The structure was a reinforced concrete structure, six storeys high with five bays in both directions, supported by pad foundations. It was designed according to British Standards and Eurocode 7 to determine the member sizes for the finite element model. Figure 1 shows the open plan structure with no facades or bracing which was used for the finite element analysis. LUSAS version 14 software package was used for the finite element analyses. Using the structural sizing from the design the finite element model was created in phases, gradually expanding the geometry. Each phase of the model was analysed, the behaviour verified and problems corrected before commencing with the next phase. Once the model was completed the effect of soil-structure stiffness was investigated by changing the soil or concrete stiffness by varying the Young’s Modulus of the material. The design of the structure and verification of the finite element model is described in detail in Smit (2010).

3 Normalisation of Data

The behaviour of a structure undergoing differential settlement is determined by its relative bending and shear stiffness, which in turn depends on the stiffness of the building materials, the geometry of the building, the geometry of the foundations and soil stiffness. For structures of this type, bending (rather than shear) stiffness is thought to be the controlling factor.

The magnitudes of column loads at ground level were used as an indicator of the behaviour of the structure. Due to symmetry within the structure only six column loads needed to be used. Figure 1 shows the location of the columns. Column A1 is the corner column, A2 and A3 are the two edge columns, B2, B3 and C3 are internal columns with C3 being the nearest to the centre of the structure. Figure 2 shows the column loads for the 5 bay structure with an imposed load of 0.5 kN/m² on the floors, an imposed line load on the edges of the floor slabs of 7.241 kN/m representing the facades, a concrete stiffness of 13 GPa and a soil stiffness that ranges from 100 Pa to 1000 GPa. The wide range of soil stiffness is unrealistic for real soils;
however it provides valuable insights into the theoretical structural behaviour. From Figure 2 it is evident that the column loads at ground level are approximately constant (but not equal) for soil stiffnesses less than 0.01 MPa and larger than 100 MPa. The column loads at ground level vary significantly over a stiffness range from 0.01 MPa to 100 MPa.

Figure 1. Structure layout.
For the same structure the stiffness of the concrete in the structure was increased by three orders of magnitude (from 13 GPa to 13 000 GPa). From the results it was evident that an increase of 3 orders of magnitude in structural stiffness is equivalent to a decrease of 3 orders of magnitude in the stiffness of the soil. It is therefore the relative bending stiffness and not the absolute values that determines the behaviour of the structure.

To determine the effect of the geometry of the building on the relative bending stiffness an ‘equivalent’ single slab with a similar stiffness was calculated. Potts and Addenbrooke (1997) have suggested two possible approaches to calculate the stiffness \((E_c I)\) of a structure. The first approach employs the parallel axis theorem to define the structural stiffness about the neutral axis as shown in Equation 1:

\[
(E_c I)_{\text{Stiffstruc}} = E_c \sum_{i=1}^{n} (I_{\text{slab}} + A_{\text{slab}} h^2)
\]  

(1)

Where \(E_c\) is the Young’s Modulus of concrete, \(I\) is the second moment of inertia, \(n\) is the number of storeys, \(A_{\text{slab}}\) is the cross sectional area of slab and \(h\) is the height. Using Equation 1 an equivalent slab thickness of 14.0 m was calculated for the model. Finite element analysis was carried out, replacing the superstructure with a single 14.0 m thick slab at ground level, with no soil contact, supported by the subsoil columns and foundations, with the same loading as the original structure, showed the replacement slab to be significantly stiffer than the structure. This can therefore be considered to be an overestimate of the building stiffness.

The alternative approach obtained the bending stiffness by adding the independent \(EI\) values of each slab as shown in Equation 2. This implies that the walls and columns transfer the same deformed shape to each storey.

\[
(E_c I)_{\text{Flexstruct}} = E_c \sum_{i=1}^{n} I_{\text{slab}}
\]  

(2)

Using Equation 2 an equivalent slab thickness of 532 mm was calculated for the model. The structural bending stiffness based on Equation 1 is approximately 18 000 times stiffer than the bending stiffness based on Equation 2. Figure 3 compares the column loads at ground level for the equivalent 532 mm slab and for the full structure. From the graph it is evident that the stiffness of the single 532 mm slab is a good approximation of the stiffness of the structure.
Based on a plane strain analysis, only, Potts and Addenbrooke (1997) defined relative bending stiffness $\rho^*$ of a building as:

$$\rho^* = \frac{EI}{E_s H^4}$$

Where $EI$ is the bending stiffness of the superstructure, $E_s$ is a representative soil stiffness and $H$ is half the width (in the plane of deformation) of the superstructure. From the equation it is evident that for a fixed building and foundation geometry $\rho^* \propto \frac{E}{E_s}$ which is supported by the finite element modelling results. From Figure 3 it is evident that $EI$ for this flexible structure without bracing or stiffening due to facades can be calculated with Equation 2. To determine the effect of building width, a 5 bay x 4 bay and a 5 bay x 3 bay model were analysed. Both models were produced by removing either 1 or 2 of the internal bays of the 5 bay model, which resulted in an identical line load on the edges of the floor slabs and identical edge and corner foundations for the structures.

Figure 4 shows the ground level column loads of the 5, 4 and 3 bay structures normalised using $\rho^*$ from Equation 4.3 where $H$ is half the length of the structure. Note that due to the formulation of $\rho^*$ the ‘stiffer soil’ is on the left of the horizontal axis, in contrast to the previous graphs where the stiffer soil is on the right. The column loads were normalised to the column load in the specific column without any soil-structure interaction effect (i.e. equivalent to being founded on an infinitely stiff soil).

Figure 4 shows that the corner column loads in the linear elastic finite element model for a ‘rigid’ structure may be up to 5 times greater than for the flexible structure. A basic load takedown underestimates the edge and corner column loads for the ‘rigid’ structure model. However, it is important to note that the results in Figure 4 are based on a linear elastic model. Yielding of the columns, floor slabs or soil would reduce the predicted column loads.

Three distinct zones of behaviour within the soil structure stiffness range can be identified within Figure 4:
- **Zone 1 ‘Flexible structure’** is the zone of relative bending stiffness ($\rho^*$) where the structure is flexible in comparison with the soil. For this structure $\rho^*$ is typically less than $1 \times 10^{-4}$ in Zone 1. The structural loads in Zone 1 can be determined without taking differential settlement into account.
Zone 2 ‘Intermediate structure’ is the intermediate zone where the loads in the edge and corner columns increase and the loads in the internal columns decrease with an increase of relative bending stiffness. For this structure $\rho^*$ typically ranges from $1 \times 10^{-4}$ to $1 \times 10^{-1}$ in Zone 2.

Zone 3 ‘Rigid structure’ is the zone of relative bending stiffness ($\rho^*$) where the structure is rigid in comparison with the soil. In Zone 3 the loads, stresses and differential movements within the structure are constant, independent of the relative bending stiffness. For this structure $\rho^*$ is typically larger than $1 \times 10^{-1}$ in Zone 3.

The designed 5 bay structure on a typical London Clay with an undrained shear strength ($S_u$) of 90 kPa results in an approximate relative bending stiffness ($\rho^*$) of $2.2 \times 10^{-6}$, which falls in Zone 1. This relative bending stiffness was based on a concrete stiffness of 13 GPa, a bending stiffness based on Equation 2, i.e. the sum of independent $EI$ values of each slab and a soil small strain stiffness of 600 MPa. The ‘typical’ structure modelled in this paper by finite element analysis will therefore behave flexibly.

The ‘typical’ structure was modelled without any internal walls or bracing to reduce the complexity. To investigate the effect of a stiffer structure the concrete stiffness was increased, instead of adding internal walls and bracing. Internal walls and bracing within structures will increase the bending stiffness of the structure. The stiffness, location and fitment details of the walls and bracing will affect the bending stiffness of the structure. The bending stiffness can be expected to be between the lower bound calculated by Equation 2 and the upper bound calculated by Equation 1. For the ‘typical’ structure modelled in this paper the bending stiffness calculated by Equation 1 is approximately 4 orders of magnitude larger than the bending stiffness from Equation 2.

4 Structural Strength

This section discusses the loads within the numerical linear-elastic models and compares them with the strength of the columns. The strengths of the columns were then compared to the loads from the linear-elastic model to indicate possible concrete failure.

Concrete, reinforcement steel and soil have non-linear stress-strain characteristics. Modelling this behaviour numerically on a full-scale structure is complex and requires significant
computing power; therefore a simplified linear elastic model was used to model the behaviour of the structure.

Because linear-elastic numerical models were used to determine the effect of differential settlement on the loads within the structure, the maximum load in the model could be infinitely high (depending on the deformation), whereas in a real building the material would yield, limiting the load. Under normal operating conditions the structural members are not intended to be stressed to failure; therefore comparing the load in the linear-elastic model to the strength gives an indication of the performance of the structure.

The loads in the numerical models are based on normal operating loads and not ultimate design loads and should therefore be significantly lower than the strength of the members.

A concrete column fails due to a critical combination of axial load and biaxial bending. To calculate and present this 3-dimensional envelope is complex and therefore the simplification as suggested in BSI 8110-1 (1997) was used. The code suggests the use of the following equations to compare biaxial bending to uniaxial bending:

\[
M_x / h' \geq M_y / b', M_x' = M_x + \beta \frac{h'}{b'} M_y
\]  

(4)

\[
M_x / h' < M_y / b', M_y' = M_y + \beta \frac{h'}{b'} M_x
\]  

(5)

Where \( h' \) and \( b' \) are the depth of the reinforcing steel and \( \beta \) is a coefficient based on the axial force on the column, the dimensions and concrete strength.

Table 1 shows the axial column loads with the equivalent uniaxial bending moment derived from the biaxial bending moments and the column loads from the model using Equations 4 and 5. As the relative bending stiffness increased in the linear elastic model the axial loads in the corner columns increased by approximately 5 times, the loads in the edge columns approximately doubled and the loads in the internal columns reduced to approximately 1/3 in comparison to the column loads within a flexible structure. The cells highlighted in grey show where the loads predicted by the linear elastic numerical model exceed the strength of the column. These column loads are based on a linear elastic model, i.e. no column, slab or foundation failure occurs. Yielding of foundations may protect the structure from damage to the columns. From Table 1 it is evident that (without foundation or slab failure) column failure may occur for a relative bending stiffness \( (\rho^*) \) higher than approximately 1.32 x 10\(^{-3} \) which is in Zone 2, intermediate stiffness. If the columns had been sized according to the expected loads calculated for a flexible structure (i.e. producing a corner column smaller than an internal column), failure in the corner and edge columns would occur at an even lower relative bending stiffness.

5 Linear vs. Non-linear Soil

To determine the effect of localised soil yielding below heavily loaded (corner and edge) columns a ‘non-linear’ model was introduced, following Atkinson (2000). For the ‘non-linear’ model, a structure with a relative bending stiffness \( (\rho^*) \) at small soil strain of 1.32 x 10\(^{1} \) (Zone 3, Rigid structure) was used.
Table 1. Predicted column failure at ground level due to axial load and bending moment.

<table>
<thead>
<tr>
<th>Column / Axial load / Bending moment</th>
<th>Zone 1 Flexible Structure</th>
<th>Zone 2 Intermediate stiffness</th>
<th>Zone 3 ‘Rigid’ structure</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1 (kN) (kN.m)</td>
<td>778</td>
<td>778</td>
<td>781</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>A2 (kN) (kN.m)</td>
<td>1596</td>
<td>1597</td>
<td>1605</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>A3 (kN) (kN.m)</td>
<td>1438</td>
<td>1441</td>
<td>1456</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>B2 (kN) (kN.m)</td>
<td>3310</td>
<td>3303</td>
<td>3267</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>B3 (kN) (kN.m)</td>
<td>3020</td>
<td>3019</td>
<td>3007</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>C3 (kN) (kN.m)</td>
<td>2733</td>
<td>2735</td>
<td>2747</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

Legend Predicted column failure

To analyse the ‘non-linear’ model a concrete stiffness of 780 000 GPa and an initial soil stiffness of 600 MPa were used. Smit (2010) discusses the derivation of the small strain soil stiffness and stiffness degradation. Below each foundation a soil block was defined to which an individual stiffness was assigned. The soil blocks were 7.5 m x 7.5 m wide and 10.4 m deep (5.4 m below foundation level) to coincide with the mesh boundaries. For the first iteration of the finite element analysis 10% of the structural load was applied and a 600 MPa soil stiffness was assigned to the soil blocks around the foundations. The settlement of each foundation was extracted from the model and a new soil stiffness calculated that allowed for stiffness degradation with strain. The calculated stiffnesses were assigned to each soil block after which the model was analysed again. These steps were repeated until the change in stiffness in each block was less than 0.1 MPa, after which the load was increased by 10% and the process repeated. Figure 5 shows the soil stiffness degradation and applied load.

![Soil Stiffness Degradation](image.png)

Figure 5. Soil stiffness degradation.
The final soil stiffnesses at 100% load were reduced to between 90 MPa and 125 MPa (for \( \rho/B \) between 0.07% and 0.13%). Table 2 compares the column loads calculated for linear and non-linear soil stiffness.

Table 2. Column loads in linear and non-linear models.

<table>
<thead>
<tr>
<th>Column</th>
<th>Column load (kN)</th>
<th>Change (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Linear soil</td>
<td>Non-linear soil</td>
</tr>
<tr>
<td>A1 (Corner)</td>
<td>3931</td>
<td>2595</td>
</tr>
<tr>
<td>A2 (Edge)</td>
<td>2954</td>
<td>2628</td>
</tr>
<tr>
<td>A3 (Edge)</td>
<td>2722</td>
<td>2424</td>
</tr>
<tr>
<td>B2 (Internal)</td>
<td>986</td>
<td>1560</td>
</tr>
<tr>
<td>B3 (Internal)</td>
<td>900</td>
<td>1481</td>
</tr>
<tr>
<td>C3 (Internal)</td>
<td>860</td>
<td>1339</td>
</tr>
</tbody>
</table>

The results show that non-linear stiffness of the soil beneath the foundations has a significant influence on the column loads.

- The load in the corner column (A1) was reduced by 34%.
- The loads in the edge columns (A2, A3) were reduced by 11%.
- The loads in the internal columns (B2, B3, C3) increased between 56% and 65%.

It is evident that the local yielding of soil around foundations may play an important role in protecting structures against column failure, and reducing the redistribution of load during soil-structure interaction.

6 Conclusions

The following conclusions can be made:

- For an elastic structure supported by a linear elastic soil it is possible to normalise the relative bending stiffness (\( \rho^* \)).
- Three distinct zones of behaviour exist within the relative bending stiffness range.
- As the relative bending stiffness increased in the linear elastic model the axial loads in the corner columns increased by approximately 5 times, the loads in the edge columns approximately doubled and the loads in the internal columns reduced to approximately 1/3 in comparison to the column loads within a flexible structure.
- The linear-elastic numerical model predicted column loads that exceeded the strength of the columns where the relative bending stiffness (\( \rho^* \)) was larger than 1.32 x 10^-3.
- Local yielding of soil around-foundations plays an important role in the redistribution of structural loads during soil structure interaction.

References


